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CONVECTION IN A POROUS LAYER FOR A TEMPERATURE DEPENDENT VISCOSITY

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Abstract—Thermal convection in a fluid-saturated porous layer is examined by means of integral relations for the case in which the viscosity is temperature dependent. The analysis deals with the high Rayleigh number limit and extends earlier work on the constant viscosity problem. A rational transformation is found which reduces the core shear distribution to a universal form.

NOMENCLATURE

$a(T_\infty)$,	function defined in equation (3.7);
$\hat{a}(T_\infty)$,	function defined in equation (3.12);
$b(T_\infty)$,	function defined in equation (3.8);
$\hat{b}(T_\infty)$,	function defined in equation (3.13);
$F(\eta)$,	boundary layer profile;
g,	acceleration due to gravity;
h,	cavity height;
$J(T_\infty)$,	function defined in equation (3.20);
K,	permeability;
k,	constant in the viscosity law;
L,	cavity aspect ratio, l/h;
l,	cavity length;
$M(T_\infty;\xi)$,	function defined in equation (3.9);
Nu,	Nusselt number;
Nu^*,	reduced Nusselt number, equation (3.25);
$Q(T_\infty)$,	function defined in equation (3.15);
R,	Darcy–Rayleigh number;
r,	fluidity;
$S(T_\infty)$,	function defined in equation (3.16);
T,	temperature;
$u(z)$,	core shear profile;
x, z,	Cartesian coordinates.

Greek symbols

α,	coefficient of thermal expansion;
β,	constant in viscosity law, equation (4.2);
δ,	boundary-layer thickness;
η,	similarity variable;
κ,	thermal diffusivity;
ν,	kinematic viscosity;
ξ,	parameter; $\xi = 0$ on $\bar{x} = 0$, $\xi = 1$ on $\bar{x} = L$;
ψ,	stream function.

Superscripts

$'$,	dimensional quantity;
$\bar{}$,	dimensionless quantity;
$\tilde{}$,	universal profile.

Subscripts

m,	value at the maximum of the stream function;
$L, 0$,	hot and cold wall values, respectively;
∞,	core values.

1. INTRODUCTION

THIS paper is concerned with buoyancy induced convection in a fluid-saturated porous layer for which the viscosity may depend strongly on the local temperature. The particular geometry of interest corresponds to a two-dimensional rectangular cavity, of finite aspect ratio, whose vertical end walls are parallel with the direction of the gravitational field. A horizontal temperature gradient is maintained across the cavity by differentially heating the end walls so that the Rayleigh number associated with the applied temperature difference is large. Problems of this type have previously been discussed, for example, by Weber [1] and, for a constant viscosity fluid, by Walker and Homsy [2] and by the present authors [3]; experimental studies have been described by Klarsfeld [4]. A comprehensive review of the literature which outlines the diversity of the applications has been given by Combarnous and Bories [5].

The steady flow that occurs at large Rayleigh numbers is characterized by a stratified inviscid core surrounded by thin thermal layers on the cavity walls. It is important to note that although the applied temperature gradient is in the horizontal direction, the core temperature field is vertically stratified. The core structure is apparently dominated by the boundary layers on the vertical end walls. Weber [1] gave an analysis of these layers, together with the compatible core structure, using the modified Oseen method described by Gill [6] for the corresponding Newtonian problem at large Prandtl numbers.

An alternative approach to cavity flows, based on integral relations, was developed by Blythe and Simpkins [7]. Application of the method to the Newtonian problem indicated that the procedure gave better results than the Oseen technique, although there was a discrepancy in the core mass flux when compared with numerical solutions of the full equations. This discrepancy was less than the corresponding result for the Oseen method, and a recent analysis [3] has shown that the disagreement was due to the inherent error in the boundary layer approximation to the full (Boussinesq) equations. In particular, the integral method [3] led to results for the mass flux, in the constant viscosity case, which were in excellent agreement with a numerical solution of the boundary-layer equations [2]. As in the Newtonian problem, the core mass flux obtained using the Oseen approach was significantly higher than that predicted by the numerical solution. A comparison with experimental data [4] for the Nusselt number and for the core temperature profile was also given in [3] for the constant viscosity case. Again the integral method led to results which were in good agreement with the observations at high Rayleigh numbers.

The analysis given in this paper extends the earlier work [3] to account for the effects of a temperature dependent viscosity. This case was first discussed by Weber [1] who used an averaged value of the viscosity for the hot and cold wall boundary layers respectively. No such approximation is made in the present paper which, as in [3], is based on an integral method. The method can take into account any desired variation of the viscosity with temperature, and results are given for the particular case of a linear fluidity–temperature law. For the present approach the only errors are those associated with the choice of velocity profiles, but for the Oseen method the averaging of the viscosity relationship introduces an additional error source. A detailed discussion of this point, and its effect, is given in Section 4.

The main finding of the present paper, however, is concerned not with the accuracy of the technique, but with the remarkable conclusion that the results in the variable viscosity case, for the core mass flux and for the core shear profile, can be collapsed onto a universal curve by means of a simple rational transformation (see Section 4). As well as suggesting a test for future theoretical calculations which use realistic viscosity models, this result should also be of great interest to experimentalists in various fields.

2. INTEGRAL RELATIONS FOR THE BOUNDARY LAYER EQUATIONS

It is convenient to use the dimensionless variables introduced in [3]. In addition it is appropriate to define a dimensionless viscosity by

$$\nu(T) = \nu'(T')/\nu'(0) \tag{2.1}$$

where primed variables have dimensions. Here T' is the temperature and, in dimensionless terms, is measured such that $T = 0$ on the cold wall $\bar{x} = 0$ and $T = L$ on the hot wall $\bar{x} = L$. All length scales are normalized with respect to the cavity height, and the lower and upper surfaces correspond respectively to $\bar{z} = 0$ and to $\bar{z} = 1$.

Near the cold wall, $\bar{x} = 0$, the boundary-layer equations reduce to [1, 3]

$$\nu(T)\frac{\partial\psi}{\partial x} = T_\infty(z) - T$$

and (2.2)

$$\frac{\partial(\psi, T)}{\partial(x, z)} = \frac{\partial^2 T}{\partial x^2}$$

where the stream function ψ is related to the dimensional quantity ψ' through

$$\psi' = \kappa R^{1/2}\psi \tag{2.3}$$

and the local length scales x and z are defined by

$$\bar{x} = R^{-1/2}x, \quad \bar{z} = z. \tag{2.4}$$

In equations (2.3) and (2.4)

$$R = \frac{K\alpha g T'_w h^2}{\kappa\nu'(0)l} \gg 1 \tag{2.5}$$

is a Darcy–Rayleigh number in which κ is the thermal diffusivity, K is the permeability, α is the coefficient of thermal expansion, g is the acceleration due to gravity, l is the cavity length, h is the cavity height and T'_w is the (dimensional) temperature difference across the cavity. Also, in equation (2.2),

$$T_\infty(z) = T(\infty, z). \tag{2.6}$$

At large Rayleigh numbers, it can be shown that in the core of the cavity the shear and temperature distributions are subject only to vertical gradients [1, 6]. Consequently

$$\psi_\infty(z) = \psi(\infty, z) \tag{2.7}$$

and $T_\infty(z)$, as well as defining the outer limit of the boundary layer solution, completely define the core stream function and the core temperature distribution. These functions must be determined from the solution of the boundary-layer equations for both end walls. For the hot wall the boundary-layer equation can be written down in a manner analogous to equation (2.2): details can be found in the paper by Weber [1].

Integration of equation (2.2) across the boundary layer, together with the corresponding results for the hot wall, implies that

$$\frac{d}{dz}\left[\int_0^\infty (T - T_\infty)^2 r(T)dx\right] \mp \psi_\infty \frac{dT_\infty}{dz} = -\left(\frac{\partial T}{\partial x}\right)_{\bar{x}=\begin{cases}0\\L\end{cases}} \tag{2.8}$$

and

$$\psi_\infty = \pm\int_0^\infty (T_\infty - T)r(T)dx \tag{2.9}$$

where

$$r(T) = \nu^{-1}(T) \quad (2.10)$$

is the dimensionless fluidity. For these equations the upper sign is associated with the cold wall $\bar{x} = 0$ and the lower sign with the hot wall $\bar{x} = L$. In the special case of constant viscosity ($\nu = 1$) the integral relations for the hot wall can be replaced by appropriate symmetry conditions [3].

It is convenient, with respect to the boundary-layer equations, to introduce the transformation

$$T \Rightarrow LT, \quad \psi \Rightarrow L^{1/2}\psi, \quad x \Rightarrow L^{-1/2}x \quad (2.11)$$

under which the general form of equations (2.6) and (2.7) is invariant. [For ease of discussion the viscosity law is still written $\nu = \nu(T)$, though strictly $\nu = \nu(LT)$ for the variables defined by equation (2.11). This law will also contain additional parameters, see Section 4.] Further, in terms of these variables, it follows that at the end walls

$$T = \frac{\partial^2 T}{\partial x^2} = \psi = 0 \quad \text{on } \bar{x} = 0 \quad (2.12)$$

and

$$T = 1, \quad \frac{\partial^2 T}{\partial x^2} = \psi = 0 \quad \text{on } \bar{x} = L. \quad (2.13)$$

Similarly, the outer limit of the boundary-layer solution is defined by

$$T \to T_\infty(z), \quad \psi \to \psi_\infty(z) \quad \text{as } x \to \infty \quad (2.14)$$

with a corresponding result for the hot wall.

3. THE CORE SOLUTION

For the cold wall boundary-layer it is assumed that the temperature profile can be approximated by the similarity form

$$T = T_\infty(z)[1 - F_0(\eta_0)] \quad (3.1)$$

where η_0 is based on the cold wall boundary-layer thickness

$$\delta_0(z) = x/\eta_0. \quad (3.2)$$

Further, F_0 satisfies the conditions

$$\text{and} \quad \left.\begin{matrix} F(0) = 1, \quad F''(0) = 0 \\ F_0(\infty) = F_0'(\infty) = \cdots = 0 \end{matrix}\right\}. \quad (3.3)$$

A corresponding assumption for the hot wall implies that

$$1 - T = (1 - T_\infty)[1 - F_L(\eta_L)] \quad (3.4)$$

where η_L is based on the hot wall boundary-layer thickness δ_L. Since $F_L(\eta_L)$ also satisfies the conditions (3.3) it is asserted that

$$F_0(\eta) = F_L(\eta) \equiv F(\eta), \quad (3.5)$$

although mathematically this assumption is not strictly necessary. Even though equation (3.5) is assumed to hold, it is emphasized that the boundary-layer thicknesses which define η_0 and η_L are not the same.

Substitution in equations (2.8) and (2.9) gives, for the cold wall layer,

$$\frac{d}{dz}[a(T_\infty)T_\infty\psi_\infty] - \psi_\infty\frac{dT_\infty}{dz} = -\frac{b(T_\infty)T_\infty^2}{\psi_\infty} \quad (3.6)$$

where

$$a(T_\infty) = M_2(T_\infty; 0)/M_1(T_\infty; 0) \quad (3.7)$$

and

$$b(T_\infty) = -F'(0)M_1(T_\infty; 0), \quad (3.8)$$

with

$$M_n(T_\infty; \xi) = \int_0^\infty F^n(\eta)r(T)d\eta. \quad (3.9)$$

In (3.9), $T = T(\eta; \xi)$ is defined by

$$T = T_\infty[1 - F(\eta)] + \xi F(\eta) \quad (3.10)$$

where $\xi = 0$ for the cold wall and $\xi = 1$ for the hot wall. The corresponding results for the hot wall layer are

$$\frac{d}{dz}[\hat{a}(T_\infty)(1 - T_\infty)\psi_\infty] + \psi_\infty\frac{dT_\infty}{dz} = \frac{\hat{b}(T_\infty)(1 - T_\infty)^2}{\psi_\infty} \quad (3.11)$$

where

$$\hat{a}(T_\infty) = M_2(T_\infty; 1)/M_1(T_\infty; 1) \quad (3.12)$$

and

$$\hat{b}(T_\infty) = -F'(0)M_1(T_\infty; 1). \quad (3.13)$$

From equations (3.6) and (3.8) it can be shown that

$$\frac{1}{\psi_\infty}\frac{d\psi_\infty}{dT_\infty} = Q(T_\infty) \quad (3.14)$$

where

$$Q(\tau) = \frac{(1 - \tau)^2\hat{b}\frac{d}{d\tau}[(1 - a)\tau] + \tau^2 b\frac{d}{d\tau}[(1 - \hat{a})(1 - \tau)]}{\tau(1 - \tau)[a\hat{b}(1 - \tau) + \hat{a}b\tau]} \quad (3.15)$$

In (3.15) the argument of the functions $a, \hat{a}, b, \hat{b}$ is to be understood as τ. Since $Q(T_\infty)$ is singular at $T_\infty = 0, 1$ it is convenient to write

$$S(\tau) = Q(\tau) - \frac{(1 - a_0)}{a_0}\frac{(1 - 2\tau)}{\tau(1 - \tau)} \quad (3.16)$$

with

$$a_0 = a(0) = \hat{a}(1). \quad (3.17)$$

$S(T_\infty)$ remains bounded as $T_\infty \to 0, 1$ and the solution of (3.14) can be put in the form

$$\psi_\infty = c[T_\infty(1 - T_\infty)]^{(1/a_0)-1} \exp\left\{\int_0^{T_\infty} S(\tau)\mathrm{d}\tau\right\} \tag{3.18}$$

where c is an arbitrary constant. For the constant viscosity case $S \equiv 0$ and equation (3.18) reduces to the result obtained in [3].

Further, it can be shown from equations (3.6) and (3.14) that

$$\frac{\mathrm{d}z}{\mathrm{d}T_\infty} = c^2 J(T_\infty) \tag{3.19}$$

where

$$J(\tau) = \frac{(c^{-1}\psi_\infty(\tau))^2}{b\tau^2}\left[1 - \frac{\mathrm{d}}{\mathrm{d}\tau}(a\tau) - a\tau Q(\tau)\right]. \tag{3.20}$$

As in earlier approaches to this problem [1, 6] it is assumed that

$$\psi_\infty(0) = \psi_\infty(1) = 0 \tag{3.21}$$

which implies from equation (3.18) that

$$T_\infty(0) = 0, \quad T_\infty(1) = 1. \tag{3.22}$$

Condition (3.21), or an equivalent statement, is necessary to complete the solution. This condition corresponds to the assertion that the vertical boundary layers empty into the core. Alternative assumptions concerning the horizontal conditions on the core solution have been made, but it was shown in [3] that the assumption (3.21) gives good results for the constant viscosity case. Strictly, the proper horizontal constraints should be derived from a solution of the boundary-layer equations which hold near the surfaces $z = 0, 1$. A recent analysis of the latter problem [8] has indicated that equation (3.21) is the correct condition in the limit $R \to \infty$. From equations (3.22) and (3.19) it follows that

$$z = \int_0^{T_\infty} J(\tau)\mathrm{d}\tau \bigg/ \int_0^1 J(\tau)\mathrm{d}\tau \tag{3.23}$$

with

$$c = \left[\int_0^1 J(\tau)\mathrm{d}\tau\right]^{-1/2}. \tag{3.24}$$

Equations (3.18), (3.23) and (3.24) provide a parametric description of the solution for a given viscosity law: $c^{-1}\psi_\infty$ can be evaluated from equation (3.18) as a function of T_∞, equation (3.20) then defines $J(T_\infty)$, and $z(T_\infty)$ is found from equation (3.23). Results for a particular choice of $r(T)$ are discussed below.

For insulated horizontal boundaries the heat transfer across the vertical planes $\bar{x} = 0, L$ is the same. An appropriate Nusselt number for the cavity heat transfer is defined by

$$Nu = L^{3/2}R^{1/2}\int_0^1 \left(\frac{\partial T}{\partial x}\right)_{x=0} \mathrm{d}z = L^{3/2}R^{1/2}Nu^* \tag{3.25}$$

where

$$Nu^* = c\int_0^1 \frac{b(\tau)J(\tau)\tau^2}{c^{-1}\psi_\infty(\tau)}\mathrm{d}\tau. \tag{3.26}$$

As noted above, equations (3.25) and (3.26) define the heat transfer across either vertical end plane only for insulated horizontal surfaces. The assumption that the horizontal boundaries are insulated does not conflict with equation (3.22) since that condition corresponds to the outer limit of the appropriate horizontal boundary-layer solution valid near $z = 0$ and $z = 1$ [8]. Independently of the horizontal thermal condition, equation (3.25) does represent the heat-transfer across the cold wall $x = 0$.

The determination of the functions a, $\hat{a}$, b and $\hat{b}$ follows from the specification of the boundary-layer profile $F(\eta)$. This profile is subject to the conditions listed in equation (3.3). Possible profiles were discussed in [3] and, as noted there, a suitable choice is the two-layer model

$$\left.\begin{aligned} F &= 1 - \frac{9}{14}\eta + \frac{1}{14}\eta^3, \quad 0 \le \eta \le 1, \\ F &= \frac{3}{7}\mathrm{e}^{-(\eta-1)}, \quad \eta > 1. \end{aligned}\right\} \tag{3.27}$$

At $\eta = 1$ the function F and its first two derivatives are continuous. The inner behavior ($\eta \to 0$) and outer behavior ($\eta \to \infty$) are both consistent with the form of the solution of the full boundary-layer equations.

4. RESULTS

Calculations were carried for the special case

$$v^{-1}(T) = r(T) = 1 + kT \tag{4.1}$$

over the range $0 \le k \le 100$. In practice k should be chosen so that equation (4.1) represents a satisfactory approximation to the viscosity variation over the dimensionless temperature interval $0 \le T \le 1$. Viscosity laws of the type in equation (4.1) are often used and an extensive discussion can be found in Partington [9]. It is not suggested that equation (4.1) will be an adequate representation of the viscosity variation when $k \gg 1$, but in this limit the results should still indicate the appropriate qualitative behavior. If a better approximation is required for liquids in which the viscosity varies very rapidly, exponential fits of the type

$$r = v^{-1} = \exp\left(\frac{kT}{1 + \beta T}\right), \tag{4.2}$$

where k and β are constants, can be used. Equation (4.2) corresponds to the dimensionless form of the Eyring model [10]. The evaluation of the functions a, b, $\hat{a}$, $\hat{b}$ etc. is a tedious task for this law and, if a more accurate representation of the data than that defined by equation (4.1) is warranted, it is easier to use polynomial fits for which equation (4.1) defines the linear approximation.

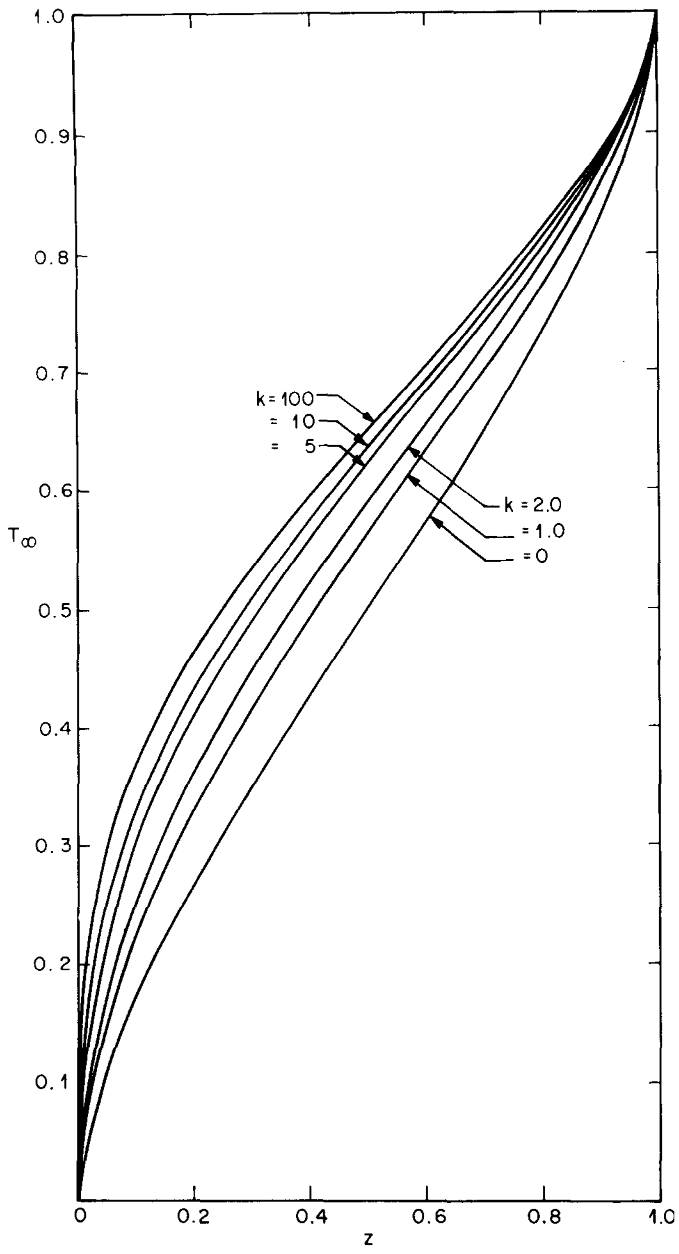

FIG. 1. Core temperature profiles for various k.

The functions $a(T_\infty)$, $\hat{a}(T_\infty)$ etc. are given in the Appendix for the viscosity law (4.1). It is assumed that the boundary-layer profile can be fitted by the two-layer model discussed in Section 3. Allowing for the singularities at the end points $T = 0$ and $T = 1$, it is then a straightforward numerical procedure to perform the quadratures associated with equations (3.14) and (3.19).

Some results for the temperature profile are shown in Fig. 1. Note that as $k \to \infty$, apart from the local behavior near $z = 0$ and $z = 1$, the temperature profile does appear to approach a limiting form. (An asymptotic analysis, $k \gg 1$, can be given confirming this result, but it does not yield a simple analytical result for the limiting profile.) The temperature profile in the cavity is often characterized by the mid-point temperature gradient which is shown in Fig. 2 as a function of k. For variable viscosity the temperature gradient is no longer a minimum at the mid-point; the minimum now occurs at $z = z_m > \frac{1}{2}$ $(k > 0)$ and is associated with the local maximum in the stream function.

The dependence of the temperature distribution on the viscosity has the same qualitative behavior as that predicted by Weber [1] using the modified Oseen method with an averaged viscosity. Figure 2 also shows the minimum temperature gradient as a

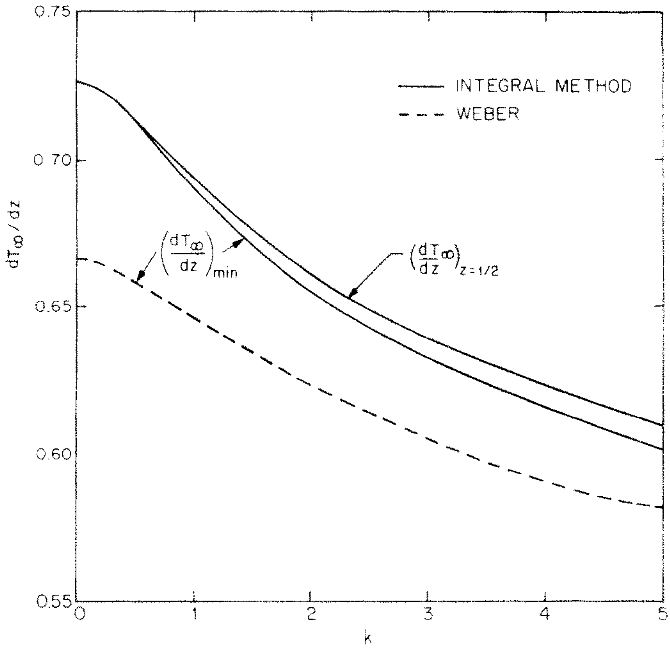

FIG. 2. Comparison of core temperature gradients.

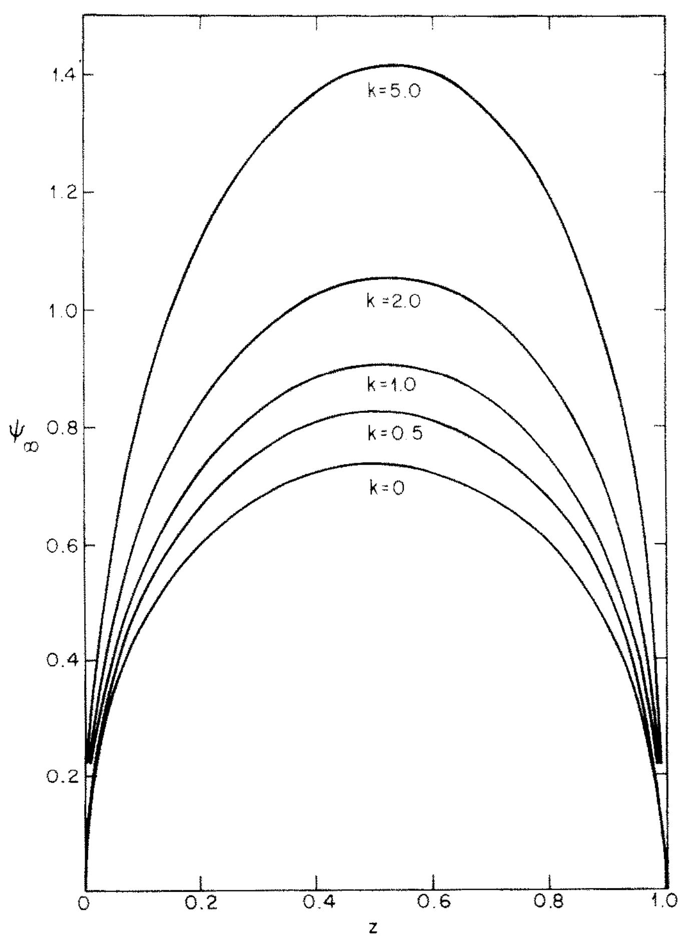

FIG. 3. Stream function profiles for various k.

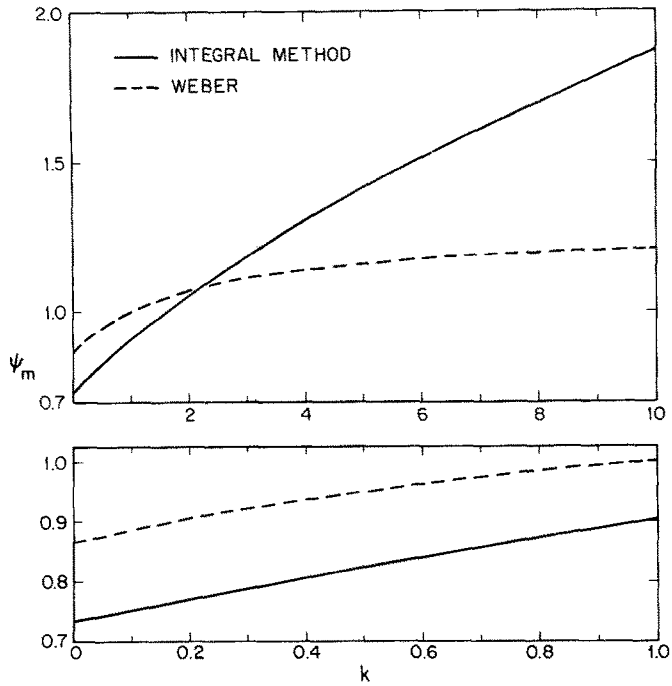

FIG. 4. Dependence of the stream function maximum ψ_m on k.

function of k for the Weber theory. As in the constant viscosity limit this characteristic core gradient lies below the results obtained by the present method. For $k = 0$ the integral method gives excellent agreement with the numerical solution of Walker and Homsy [2]. As $k \to \infty$ the Oseen value of $(dT_\infty/dz)_{min} \to 0.509$, compared with the value 0.535 predicted by the present theory.

Stream function profiles are shown in Fig. 3. The dependence of the stream function maximum ψ_m on the viscosity parameter k is displayed in Fig. 4 both for the present integral method and for the Oseen (averaged viscosity) approach. As noted earlier, for $k = 0$ the integral method agrees well with a numerical solution of the boundary-layer equations, but the Oseen approach leads to a significant overestimate of the core mass flux. For moderate values of k ($\lesssim 1$) this behavior is repeated and the two solutions have a similar trend with respect to k. At larger values of k, however, a marked change in this pattern occurs. The Oseen approach, using an averaged viscosity, leads to

$$\psi_m \to 1.277, \quad k \to \infty, \tag{4.3}$$

while for the present integral method

$$\psi_m \sim 0.553k^{1/2}, \quad k \to \infty. \tag{4.4}$$

In the dimensionless units used here the cold wall viscosity $\nu(0) = 1$ and, as $k \to \infty$, the hot wall viscosity $\nu(1) = O(k^{-1})$. Further, away from any region where $T \ll 1$, i.e. the cold wall and the bottom of the core, the viscosity is $O(k^{-1})$ everywhere. This suggests that the hot wall viscosity is a more characteristic value on which to base the Rayleigh number when $k \gg 1$. Introduction of this change suggests that $\psi_m k^{-1/2}$ should limit as $k \to \infty$ [see equations (2.3) and (2.5)], which is in accord with the theoretical result, equation (4.4). It is apparent that the averaging approach fails for large k. For the Oseen method, as $k \to \infty$, the averaged cold wall boundary-layer viscosity is 3/4, and the corresponding value for the hot wall layer is 1/4 [1]. This should be contrasted with the observation made above that, over most of the region, the effective viscosity is $O(k^{-1})$.

In order to display the present results for the stream function over a broad range of the viscosity parameter, it appeared to be most convenient to use as dependent variable ψ_∞/ψ_m. Inspection of the calculations further indicated that the results could possibly be collapsed onto a universal curve of the form

$$\frac{\psi_\infty(z;k)}{\psi_m(k)} = \tilde{\psi}_\infty(\tilde{z}) \tag{4.5}$$

where

$$\tilde{z} = \frac{(1-z_m)z}{z - z_m + 2z_m(1-z)}. \tag{4.6}$$

The choice of the rational transformation (4.6) is such that for $k = 0$ (i.e. $z_m = \frac{1}{2}$), $\tilde{z} = z$. Further, for all k,

$$\left.\begin{aligned} \tilde{z} &= 0 \quad \text{at } z = 0, \\ \tilde{z} &= \tfrac{1}{2} \quad \text{at } z = z_m, \\ \tilde{z} &= 1 \quad \text{at } z = 1. \end{aligned}\right\} \tag{4.7}$$

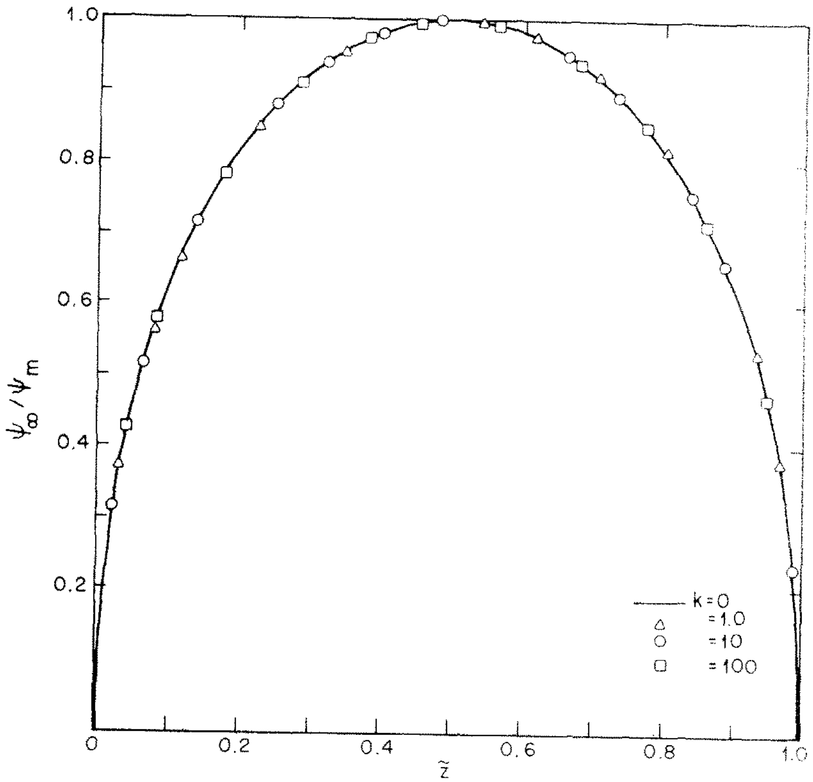

FIG. 5. The universal curve for the reduced stream function.

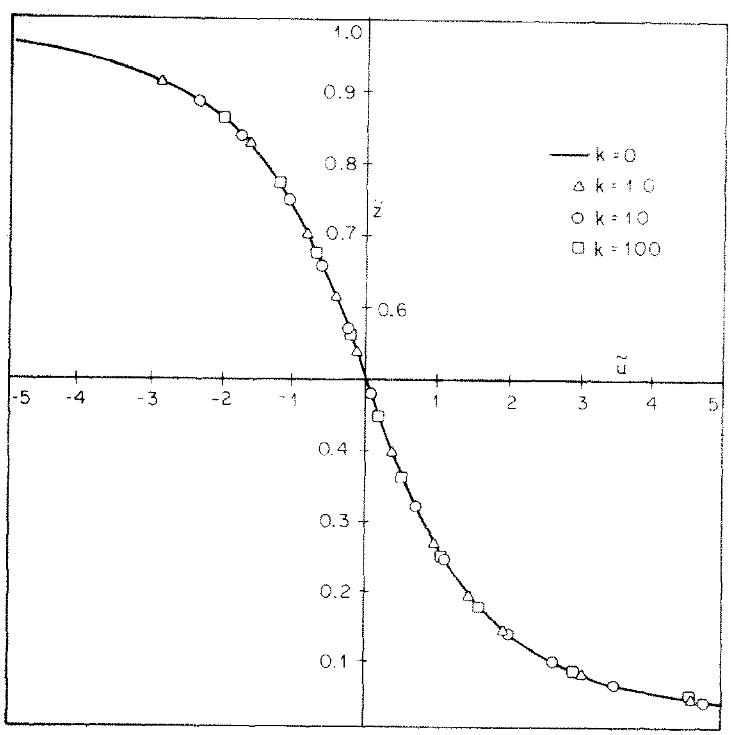

FIG. 6. The universal curve for the reduced shear profile.

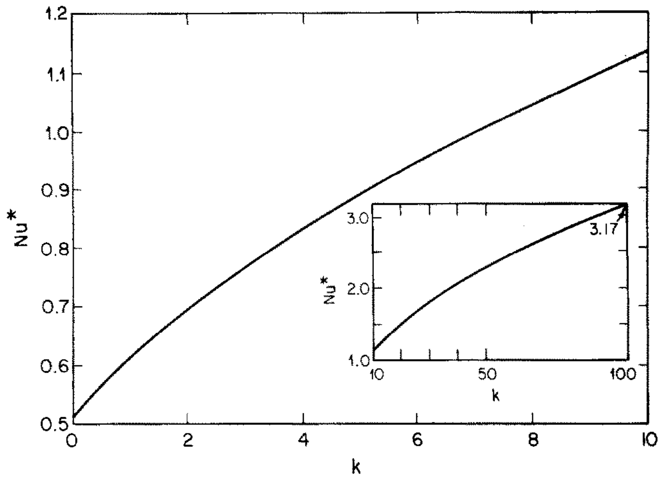

FIG. 7. Scaled Nusselt number $Nu^*(k)$.

Obviously, if a universal curve exists it is defined by

$$\tilde{\psi}_\infty(\tilde{z}) = \frac{\psi_\infty(\tilde{z};0)}{\psi_m(0)}. \tag{4.8}$$

This function, together with results for $k = 1$, 10 and 100, is shown in Fig. 5. It is apparent that the results are well fitted by the profile (4.8). Equation (4.5) can be differentiated to give a reduced shear profile

$$\tilde{u}(\tilde{z}) = \frac{d\tilde{\psi}_\infty}{d\tilde{z}} = \left(\frac{1-z_m}{z_m}\right)\left(\frac{\tilde{z}}{z}\right)^2 \frac{1}{\psi_m}\frac{d\psi_\infty}{dz}. \tag{4.9}$$

Again if $\tilde{u}(\tilde{z})$ does reduce to a universal curve it must correspond to the solution defined by the constant viscosity profile for $k = 0$. Results for $k = 0$, 1, 10 and 100 are shown in Fig. 6. Although the differential form (4.9) is a stronger test of the scaling law than the stream function profiles, the data again collapses remarkably well onto the single curve corresponding to $k = 0$.

The heat transfer across the cavity is defined by the Nusselt number

$$Nu = L^{3/2}R^{1/2}Nu^*(k) \tag{4.10}$$

when Nu^* is defined by (3.26). Results for $Nu^*(k)$, for the viscosity law (4.1), are shown in Fig. 7.

The heat transfer across the cavity, both for Weber and for the present theory, is characterized by a Nusselt number–Rayleigh number relation of the form

$$Nu \sim CR^{1/2}, \quad R \to \infty$$

where the constant C depends on the cavity aspect ratio L and the parameter k associated with the viscosity–temperature law. In neither theory does the viscosity variation affect the $R^{1/2}$ law ($R \to \infty$) but merely changes the constant of proportionality C ($= L^{3/2}\ Nu^*(k)$) independently of R. Numerical solutions (Bankvall, [12]) confirm this law in the constant viscosity case as $R \to \infty$. Other theories which attempt to take into account departures from this law at smaller values of R have been proposed (see Bejan [13]). In all cases these laws still reduce to the above form as R increases. A detailed discussion was given in Section 5 of the first paper [3].

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APPENDIX

The functions $a(T_\infty)$, $\hat{a}(T_\infty)$, $b(T_\infty)$ and $\hat{b}(T_\infty)$

For the viscosity law (4.1) these functions are given by

$$\left.\begin{aligned} a &= \frac{l_2 + (l_2 - l_3)kT_\infty}{l_1 + (l_1 - l_2)kT_\infty}, \\ \hat{a} &= \frac{l_2 + l_3 k + (l_2 - l_3)kT_\infty}{l_1 + l_2 k + (l_1 - l_2)kT_\infty}, \end{aligned}\right\} \quad (A1)$$

and

$$\left.\begin{aligned} b &= -F'(0)[l_1 + (l_1 - l_2)kT_\infty], \\ \hat{b} &= -F'(0)[l_1 + l_2 k + (l_1 - l_2)kT_\infty], \end{aligned}\right\} \quad (A2)$$

where

$$l_n = \int_0^1 F^n(\eta)\,d\eta. \quad (A3)$$

For the inner–outer profile described in Section 3

$$l_1 = \frac{9}{8}, \quad l_2 = \frac{4149}{6860}, \quad l_3 = \frac{6633}{15680} \quad (A4)$$

and

$$F'(0) = -\frac{9}{14}. \quad (A5)$$

CONVECTION DANS UNE COUCHE POREUSE POUR UNE VISCOSITE FONCTION DE LA TEMPERATURE

Résumé — On étudie la convection thermique dans une couche poreuse saturée de fluide, à l'aide de relations intégrales dans le cas où la viscosité est fonction de la température. L'analyse envisage la limite du nombre de Rayleigh et elle prolonge les travaux antérieurs relatifs à une viscosité constante. Une transformation rationnelle réduit la distribution centrale à une forme universelle.

KONVEKTION IN EINER PORÖSEN SCHICHT BEI TEMPERATURABHÄNGIGER ZÄHIGKEIT

Zusammenfassung—Thermische Konvektion in einer flüssigkeitsgesättigten porösen Schicht wird mit Hilfe integraler Beziehungen für den Fall temperaturabhängiger Zähigkeit untersucht. Die Untersuchung behandelt den Grenzbereich hoher Rayleigh-Zahlen und setzt frühere Arbeiten zum Problem konstanter Zähigkeit fort. Es wurde eine zweckmäßige Transformation gefunden, mit deren Hilfe die Schubspannungsverteilung im Zentrum auf eine universelle Form gebracht werden kann.

КОНВЕКЦИЯ В ПОРИСТОМ СЛОЕ С ВЯЗКОСТЬЮ, ЗАВИСЯЩЕЙ ОТ ТЕМПЕРАТУРЫ

Аннотация — С помощью интегральных соотношений исследуется тепловая конвекция в пористом слое, насыщенном жидкостью, для случая, когда вязкость зависит от температуры. Рассматривается область высоких чисел Релея, и результаты ранее проведенной работы обобщаются на случай постоянной вязкости. Получено преобразование, позволяющее привести напряжение сдвига в центре слоя к универсальному виду.